Math 205 HW 8/31

1. Prove the following claim about any two sets A and B: $A \subseteq B$ iff $A \cap B = A$.

2. Let $f : A \to B$ be a function between two sets, and let $A_0 \subseteq A$ and $B_0 \subseteq B$. Decide whether each of the following four claims is true or false. If true, provide a proof. If false, provide an actual counterexample:

   (a) $A_0 \subseteq f^{-1}(f(A_0))$:

   (b) $A_0 \supseteq f^{-1}(f(A_0))$:

   (c) $B_0 \subseteq f(f^{-1}(B_0))$:

   (d) $B_0 \supseteq f(f^{-1}(B_0))$:

3. First, some definitions. A function $f : A \to B$ is called *injective* if $f(a) = f(a')$ implies $a = a'$ (that is, no two distinct elements of A get mapped to the same element of B). A function $f : A \to B$ is called *surjective* if $f(A) = B$ (that is, every element of B is hit by at least one element of A). Finally, f is called *bijective* if it is both injective and surjective:

   (a) Let $\mathbb{N} = \{1, 2, 3, \ldots\}$ denote the set of positive integers. Find a function $f : \mathbb{N} \to \mathbb{N}$ which is injective but not surjective:

   (b) Construct a bijective function $g : \mathbb{N} \to \mathbb{N} \times \mathbb{N}$:

   (c) Suppose $f : A \to B$ is bijective. Show that there exists a function $g : B \to A$ such that $(g \circ f)(a) = a$ for every $a \in A$, and $(f \circ g)(b) = b$ for every $b \in B$: