

1. Do problem 6 (only parts (a) and (b)) from Section 2.2 in “Topology: A first course”

2. Let (X, \mathcal{T}) and (Y, \mathcal{T}') be topological spaces. A function $f : X \rightarrow Y$ is called *continuous* if $f^{-1}(U') \in \mathcal{T}$ for every $U' \in \mathcal{T}'$. (in words: if the preimage of every open set is an open set).

(a) Suppose \mathcal{T}' is generated by the basis \mathcal{B}' . Prove that f is continuous if and only the following condition holds: $f^{-1}(B') \in \mathcal{T}$ for every $B' \in \mathcal{B}'$ (in other words, show that you only need to check continuity on a basis!):

(b) Let X be a set with the discrete topology. Characterize all continuous functions $f : X \rightarrow \mathbb{R}$, where \mathbb{R} has the usual topology.

(c) Let X be a set with the indiscrete topology. Characterize all continuous functions $f : X \rightarrow \mathbb{R}$, where \mathbb{R} has the usual topology.