1. Let X be a set and Y a subset of X. Let $\mathcal{T}$ and $\mathcal{T}'$ be two topologies on X, and let $\mathcal{T}_Y$ and $\mathcal{T}'_Y$ be the topologies they induce on Y. Suppose $\mathcal{T}'$ is strictly finer than $\mathcal{T}$.

   (a) Prove that $\mathcal{T}'_Y$ is finer than $\mathcal{T}_Y$.

   (b) Show by explicit counterexample that $\mathcal{T}'_Y$ need not be *strictly* finer than $\mathcal{T}_Y$

2. Let X and Y be topological spaces, and give $X \times Y$ the product topology. Consider the projection function $\pi : X \times Y \to X$ given by the formula $\pi(x, y) = x$. Is this a continuous function? If yes, prove it. If not, find an explicit counterexample.

3. Show that every subspace Y of a Hausdorff space X is itself a Hausdorff space.