Math 205 **HW 9/9**

1. Let X be an arbitrary topological space.

   (a) For any two sets $A, B \subseteq X$, prove that $\overline{A \cup B} = \overline{A} \cup \overline{B}$:

   (b) For any arbitrary collection of sets A_α, prove the inclusion

   $$\bigcup_\alpha \overline{A_\alpha} \subseteq \overline{\bigcup_\alpha A_\alpha} :$$

   (c) Show by explicit counterexample that the reverse inclusion is not always true:

2. Given a subset A of a top. space X, we define the *boundary* of A to be $Bd(A) = \overline{A} \cap \overline{A^c}$.

   (a) Show that the interior of A and the boundary of A are always disjoint.

   (b) Show that the closure of A equals the union of its interior and its boundary.

   (c) Show that $Bd(A) = \emptyset$ if and only if A is both closed and open.