1. Let X be a topological space, and give $X \times X$ the product topology. Define the *diagonal* $\Delta_X = \{(x, x) \mid x \in X\}$ Show that X is Hausdorff if and only if Δ_X is closed in $X \times X$:

2. Let $X = (0, 1)$ and $Y = (0, \infty)$, both considered to be subspaces of $\mathbb{R}$ with the usual topology. Construct a homeomorphism $f : X \to Y$. Make sure to show that it is a homeomorphism: