

1. Let $f : X \rightarrow \mathbb{R}$ and $g : X \rightarrow \mathbb{R}$ be two continuous functions, where \mathbb{R} has the usual topology and X is an arbitrary topological space.

(a) Given $a \in \mathbb{R}$, the *level set* of f at a is defined to be $X_a = \{x \mid f(x) = a\}$.
Prove that X_a is closed in X , for all $a \in \mathbb{R}$:

(b) Prove that the set $\{x \mid f(x) \geq g(x)\}$ is closed in X :

2. Let A, B, C, D be topological spaces, and suppose $f : A \rightarrow B$ and $g : C \rightarrow D$ are continuous functions. Define the function $f \times g : A \times C \rightarrow B \times D$ by the rule $(f \times g)((a, c)) = (f(a), g(c))$. Show that $f \times g$ is a continuous function.