

1. Characterize all convergent sequences in a set X with the discrete topology (that is, make a claim like “the sequence (x_n) converges iff ”something,“ and then prove your claim.).

2. Given two points $x = (x_1, x_2, \dots, x_n)$ and $y = (y_1, y_2, \dots, y_n)$ in \mathbb{R}^n , define $d(x, y) = |x_1 - y_1| + |x_2 - y_2| + \dots + |x_n - y_n|$.

(a) Show that d is a metric on \mathbb{R}^n :

(b) Show that d induces the usual topology on \mathbb{R}^n :

(c) Sketch a typical basis element when $n = 2$: