Math 205 HW 9/21

1. Let $I = [0,1]$, and define $X = I \times I$ to be the unit square (with the subspace topology inherited from the Euclidean plane). Define an equivalence relation $\sim$ on X by declaring $(x,y) \sim (s,t)$ if $(x,y) = (s,t)$ or if $x = s$ and $y, t \in \{0,1\}$. Let $Y = X/\sim$ with the quoitent topology.

   (a) Using several copies of X as your drawing board, shade in all the possible typical open sets of Y:

   (b) Prove that Y is homeomorphic to the cylinder $Z = S^1 \times I$ with the subspace topology inherited from Euclidean three-space:

2. Let X be a topological space and let A be a *set*, with $p : X \to A$ a surjective function. Show that the quotient topology induced by p is the finest topology on A relative to which p is continuous:

3. Let $Z = (\mathbb{R} \times \{0\}) \cup (\{0\} \times \mathbb{R})$, considered to be just a set for the moment. Define $g : \mathbb{R}^2 \to Z$, where $\mathbb{R}^2$ has the standard topology, by the rule $g(x, y) = (x, 0)$ if $x \neq 0$, and $g(x, y) = (0, y)$ if $x = 0$. Show that Z is not Hausdorff if given the quotient topology induced by g.