Math 205 HW 9/23

1. Is $\mathbb{R}_\ell$ (the real numbers with the lower-limit topology) connected or disconnected? Prove your answer:

2. Let $A_1, A_2, \ldots, A_N$ be a finite sequence of connected subsets of a top. space X such that $A_k \cap A_{k+1} \neq \emptyset$, for all $1 \leq k \leq N-1$. Show that $Y = A_1 \cup A_2 \cup \ldots \cup A_N$ is also connected:

3. A topological space X is called *totally disconnected* if all of its connected components are one-point sets. For each of the following statements, provide either a proof or an explicit counterexample:

   (a) If X has the discrete topology, then X is totally disconnected:

   (b) If X is totally disconnected, then X has the discrete topology: