Math 205 HW 9/26

1. Let $f : X \to X$ be a continuous function from a topological space to itself. A *fixed point* of f is a point $x \in X$ such that $f(x) = x$.

   (a) Let $X = (0, 1)$. Construct a continuous function from X to itself with no fixed point:

   (b) Let $X = [0, 1]$. Show that every continuous function from X to itself must have a fixed point:

2. A topological space (X, T) is said to be *locally path-connected* if T has a basis of path-connected sets. Fix an $x \in X$, and define $P_x \subset X$ to be the *path component* of x: the set of points $y \in X$ such that there exists a path in X from x to y. Now prove the following statements:

   (a) If X is locally path-connected, then P_x is open in X:

   (b) If X is locally path-connected, then P_x is closed in X:

(c) X is path connected iff X is both connected and locally path-connected:

3. Show that $\mathbb{R}^n$ $(n > 1)$ is not homeomorphic to $\mathbb{R}$: