

1. Is \mathbb{R}_ℓ (the real numbers with the lower-limit topology) compact or not? Prove your answer either way:

2. Show that a finite union of compact sets is compact:

3. Let X and Y be topological spaces, and give $X \times Y$ the product topology. Recall that the projection $\pi_1 : X \times Y \rightarrow X$ is defined by $\pi_1(x, y) = x$.

(a) Prove: if Y is compact, then π_1 is a closed map (meaning: A closed in $X \times Y$ implies $\pi_1(A)$ closed in X).

(b) Show via explicit counterexample that the claim fails when Y is not compact: